Smoothing the Volatility Smile using the Corrado-Su Model

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Abstract
The expansion of the derivatives market both globally and particularly in Brazil has driven users to enhance and develop tools for more efficient pricing. However, despite this expansion of the derivatives market, certain characteristics of the Brazilian options market still impose limits on its analysis and thus the studies performed. Despite the most commonly used pricing model in Brazil being that of the Black-Scholes, its assumption of constant asset’s volatility up to maturity and log-normal distribution price are criticized by a number of authors, because those are not sustained in real-life situations and consequently leads to inconsistencies. The objective of the model suggested by Corrado and Su, and improved by Brown and Robinson, is to adapt the BS model to the distribution of the asset intended for pricing by introducing the returns’ kurtosis and skewness. For this reason, it is expected that the model will be more apt for calculating implied volatility in order to reduce the volatility smile. The purpose of this paper is to show which window of observation generates the kurtosis and skewness which smoothes the volatility smile most, using the Corrado-Su model. Therefore, the companies chosen for the study were Petrobrás PN and Vale PNA, because their stocks and options are the most liquid on the Brazilian market. The data analysis indicated a smoother volatility smile using short term observation windows rather than long term windows and a performance of earlier windows equivalent to those of the Black-Scholes model.

Key words: Option pricing; Corrado-Su; implied volatility; volatility smile
1. Introduction

The Brazilian derivatives market has been growing steadily, as well as its importance to investors and companies due to the fact that it adds value to the organizations that use it (ROSSO JUNIOR, 2008). As a consequence of this market expansion, users are motivated to look for more efficient pricing models.

However, despite this expansion of the derivatives market, certain characteristics of the Brazilian options market still impose limits on its analysis and thus the studies performed. Some of these characteristics are: the lack of liquidity of options with expiration dates of more than three months, the reduced liquidity of put options and the small number of firms that have highly liquid short-term call options, with liquidity concentrated in Petrobras and Vale options.

Despite the most commonly used pricing model in Brazil being that of the Black-Scholes (BS) (BARBACHAN and ORNELAS, 2003; BESSADA, BARBEDO and ARAÚJO, 2007), its assumption of constant asset’s volatility up to maturity and log-normal distribution price (BLACK and SCHOLES, 1973) are criticized by a number of authors (NATENBERG, 1994; VÄHÄMAA, 2003; HENTSCHEL, 2003; HULL, 2008), because those are not sustained in real-life situations and consequently leads to inconsistencies.

The volatility smile is one of this inconsistencies which has been studied extensively. Hull (2008) posits that this incongruity exists as a consequence of the asset’s distribution being different to that used by the BS model. Despite this, because it is easy and simple to use, the BS has become the most widely used model for calculating implied volatility (MCDONALD, 2006; CHARGOY-CORONA and IBARRA-VALDEZ, 2006).

The objective of the model suggested by Corrado and Su (1996) and improved by Brown and Robinson (2002) is to adapt the BS model to the distribution of the asset intended for pricing by introducing the returns’ kurtosis and skewness. For this reason, it is expected that the model will be more apt for calculating implied volatility in order to reduce the volatility smile. A León, Rubio and Serna (2005) study suggests that adjusting the distribution using kurtosis and skewness is efficient for calculating volatility, since the authors adjusted a GARCH model while using these parameters to calculate volatility and obtained consistent results.

Focusing on these facts, this paper attempted to show which data observation window supplies the kurtosis and skewness which most effectively smoothes the volatility smile of Brazilian assets. This reduction is measured by comparing the slope coefficients of the volatility curves calculated.

This study is considered relevant due to the criticisms of the BS model for generating data with biases and inconsistencies (BERTUCCI, 1999; HENTSCHEL, 2003; TABAK and GUERRA, 2007), which is linked to the importance of calculating volatility for pricing options and other analyses as undertaken by Tabak and Guerra (2007) when verifying a correlation between the return of Brazilian assets and the current and future volatility.

Studies undertaken to emphasize the importance of this specific topic are scarce as they normally calculate future volatility by applying forecasting models using past data (SANTOS and ZIEGELMANN, 2012; GOULART and others, 2005; PENZER, WANG and YAO, 2009; CHARLES, 2008; TABAK and GUERRA, 2007; BERTUCCI, 1999; KLAASSEN, 2002). Additionally, the Brazilian market also presents specific peculiarities, such as: stronger impact of negative events on volatility, as shown by Ceretta and Costa Jr. (2001); higher market volatility, which provokes an increase of the volatility smile, as shown by Lanari and Souza (2000), all of which contribute to making this paper a relevant and necessary study.

The study also sought to contribute to a better understanding of the Brazilian options market insofar as it attempts to assess whether its approach, which has not been adopted in
any other Brazilian study, is pertinent to the issue at hand. Thus, we will better understand the
dynamics of Brazilian options and whether the use of the proposed model can open up new
possibilities.

In order to achieve the proposed objective, an empirical-analytical study was
undertaken, which according to Martins (2007), is a methodology normally used in
quantitative research because it favors practical studies, instruments of measurement and
degrees of reliability when assessing the proposed hypotheses.

The results obtained point to a volatility smile that is smoother in the short term data
windows as opposed to the long term windows and a performance of the earlier equivalent to
the Black-Scholes model.

These results therefore question the use of the Corrado-Su model for smoothing the
volatility smile and highlight the advantages of using the Black-Scholes model, given that it
needs smaller amounts of market data and thus fewer research and computational resources.

Introduction excluded, this paper is organized as follows: the first section presents the
theoretical references regarding the proposed topic, the second presents the methodology
applied, the results of analyses are described in the third and the conclusions expose in the
last section.

2. Theoretical Reference
2.1. Brazilian Derivatives Market

As the name suggest, derivatives are financial instruments which depend on other,
more basic variables, in such a way that their values are derived from other assets or
contracts. (HULL, 2008; FIGUEIREDO, 2010; BESSADA, BARBEDO and ARAÚJO,
2007).

In the Brazilian market call options are more heavily traded than put options. In
addition, call options on shares traded on the BM&FBovespa exchange are not, as a whole,
very liquid, with Petrobras and Vale options currently concentrating most of the market’s
liquidity.

However, even in the case of these two companies’ options, the only highly liquid ones
are those closest to the expiration dates, which always takes place on the third Monday of the
month. As there are not many, usually only two, expiries with liquidity, it is difficult to create
a volatility surface that understands the dynamics and characteristic of this market.

In addition, the research did not verify a reasonable number (more than seven) of
options with the same expiration date with a high level of liquidity, a fact that affects the
evaluation of the volatility smile, this study’s main theme.

These characteristics of the Brazilian market will have a direct impact on the present
study, an issue which is discussed in the methodology section.

2.2. Options

An option is an agreement which provides the right to sell or purchase an asset
according to pre-defined conditions (BLACK and SCHOLEs, 1973), of which there are two
kinds: purchase and sale options, known as call and put respectively (HULL, 2008;

Options are also classified according to the moment at which their purchase or sale
can be exercised. European options concede this right only on the option’s maturity date, the
date on which the conceded right expires; whereas American options can be exercised at any
moment up to maturity (HULL, 2008; FIGUEIREDO, 2010; BLACK and SCHOLEs, 1973).
Call options are classified according to their exercise price (E) in relation to the cash price (S) at the moment of signature of the agreement. If E is less than S, the option is said to be in-the-money, if E is equal to S, it is known as at-the-money and if E is more than S, it is out-of-the-money (FIGUEIREDO, 2010). This classification, specifically, is relevant in this paper because it will be widely used to assess the volatility smile.

The price of these options is affected by five variables. Namely: the asset’s cash price (S), the exercise price (E), the risk free interest rate (r), the time until maturity (t) and the asset’s volatility (σ). According to Hull (2008) and FIGUEIREDO (2010), the dividends paid will be the sixth variable to affect them because they will provoke a reduction in the asset price. However, this impact does not occur on the Brazilian market because the exercise price is also reduced by the value of the dividend, protecting them from this impact (BESSADA, BARBEDO and ARAÚJO, 2007).

2.3. Volatility
Volatility is a measurement of uncertainty regarding future returns of a given stock (HULL, 2008), it measures the dispersion of expected returns (BESSADA, BARBEDO and ARAÚJO, 2007). Because it is not directly visualized on the market and varies over time, it has become an important input for derivatives pricing models (MCDONALD, 2006; NATENBERG, 1994).

Volatility will indicate the degree of probability that the asset will reach a specific value in the future (BESSADA, BARBEDO and ARAÚJO, 2007). Faced with the impossibility of obtaining the future volatility, it must therefore be estimated and there are a number of models to do this. The model chosen for the calculation of the volatility was the implied volatility.

2.4. Implied volatility
Because volatility is a parameter that is not observed on the market, specific methods are used to forecast it, as explained above. In addition to forecasting, it is also possible to find which volatility, together with the other parameters observed, reached the option’s market trade price. This volatility is called implied volatility (HULL, 2008; BESSADA, BARBEDO and ARAÚJO, 2007; MCDONALD, 2006, NATENBERG, 1994).

The Black-Scholes model is normally used to calculate implied volatility (MCDONALD, 2006). Because it does not perfectly adhere to real situations, different volatilities are obtained for the same asset in accordance with the variance of the options’ exercise price, maintaining all other parameters constant (HULL, 2008; MCDONALD, 2006).

The volatility smile is visualized by creating a price graph for the exercise (X axis) using the asset’s implied volatility (Y axis) (BEFFADA, BARBEDO and ARAÚJO, 2007; MCDONALD, 2006; LANARAI and SOUZA, 2000).

Despite the fact that the graph explained above is usually convex, (the reason why it was given the name smile), this graph can be presented in another way when there is staggered volatility provoking decreased or increased values depending on whether the exercise price is in or out of the money.

McDonald (2006) outlines the advantages of using implied volatility. The first is to validate a given pricing model, because the lower the smile, the better the model. Secondly, instead of the price of the option itself, volatility often serves as a value measurement for the markets. Thirdly, it serves as a volatility expected into the future by the market. Fourth, the model is a fast and easy way to obtain the volatility of a given asset and then apply that
volatility to another similar asset if it has no data regarding price or volatility available on the market.

Banerjee, Doran and Peterson (2007) complement these advantages. In their study they showed that the implied volatility measured by the American VIX index market, (the index which represents the volatility expected by the American market for S&P 100 stocks over the next 30 days), present a high correlation with the diversified portfolios, showing that it is possible to forecast futures returns using implied volatility.

In addition, Gabe and Portugal (2004) study the higher explanation power of future volatility using implied volatility because it is based on \textit{ex-ante} parameters, contrary to those models based on \textit{ex-post} parameters, such as GARCH, despite the fact this was not proven in their study.

Galvão (2002) emphasizes Gabe and Portugal’s (2004) results when he compared the models for pricing options and concluded that implied volatility performed better.

### 2.5. \textbf{Corrado-Su Model}

The Corrado-Su model is based on an expansion of the Black-Scholes formula in order to incorporate non-normal distributions, where kurtosis and skewness of the assets’ returns are considered (CORRADO and SU, 1996). To do so, the authors use the expansion of the Gram-Charlier normal probability density series to model the distribution of the asset’s return.

The formula used by Corrado and Su (1996) contained two typographical errors, which were observed and corrected by Brown and Robinson (2002).

Later, Jurczenko, Maillet and Negrea (2004) slightly modified the model in order to prove consistency with a Martingale restriction.

Despite the differences between the results of the original Corrado-Su model and the correct and modified ones being mostly very slight, it may be economically significant for specific cases such as options with a far-off maturity date or those that are deeply out of the money, in particular when the market is turbulent (JURCZENKO, MAILLET and NEGREA, 2004). Because the Corrado-Su model modified by Jurczenko, Maillet and Negrea (2004) is more accurate, this will be used to calculate implied volatility in this paper.

The difference between the formulas of the Black-Scholes model and those of the Modified Corrado-Su model is the addition of two terms to the first model, which added the analyzed series’ skew and kurtosis parameters. The equation of the model which defines the option purchase price is presented below:

\[
\text{Call Price} = S \times N(d_1) - E \times \exp(-r \times T) \times N(d_2) + \mu_3 \times Q_3 + (\mu_4 - 3) \times Q_4 \quad (1)
\]

In that:

\[
Q_3 = \frac{1}{6(1+w)} \times S \times \sigma \times \sqrt{T} \times (2\sigma \times \sqrt{T} - d_1) \times N(d1) \quad (2)
\]

\[
Q_4 = \frac{1}{24(1+w)} \times S \times \sigma \times \sqrt{T} \times \left( d1^2 - 3d1 \times \sigma \times \sqrt{T} + 3\sigma^2 \times T - 1 \right) \times N(d1) \quad (3)
\]

\[
w = \frac{\mu_3}{6} \times \sigma^3 \times T^{1.5} + \frac{\mu_4}{24} \times \sigma^4 \times T^2 \quad (4)
\]

\[
d1 = \frac{\ln(S/E) + (r + \frac{\sigma^2}{2}) \times T}{\sigma \times \sqrt{T}} \quad (5)
\]
\[ d_2 = d_1 - \sigma \times \sqrt{t} \]  

(6)

Where:

- \( \exp(x) \) – the exponential of the variable \( x \);
- \( N(x) \) – is the cumulative probability function of the standard normal variable \( x \);
- \( \mu_3 \) – skew coefficient;
- \( \mu_4 \) – kurtosis coefficient.

3. Methodology

The purpose of this study is to show which data observation window provides the kurtosis and skewness which most reduces the volatility smile of Brazilian assets in the Corrado-Su model.

To achieve the proposed objective, an empirical-analytical study was undertaken. According to Martins (2007) this methodology is normally used in quantitative research because it favors practical research, measuring instruments and reliability levels when verifying the proposed hypotheses. To provide theoretical foundation to the empirical research, a bibliographical study was performed in order to, as stated by Vergara (2009), position the reader in the topic and justify the procedures used by the author to achieve the study’s objective.

After the bibliographical research and a comprehension of the study topic was obtained, the Brazilian companies to be included in the study were defined. Petrobrás PN and Vale PNA were chosen due to the fact that their stocks and call options are the most liquid on the Brazilian market, which is essential for the study’s importance and validity. Given that a lack of liquidity could generate problems such as inconsistent results and the impossibility of generalizing them, among others.

Following this, the closing prices of the stocks mentioned were collected from the Economática database, in original currency and adjusted by dividends and other bonus, eliminating the days on which there was no trading, so that the calculation of kurtosis and skewness was not influenced by the dividends and other bonus.

With the Economática data collected, the daily return of the stocks was calculated according to the formula \( r_t = \ln \left( \frac{P_t}{P_{t-1}} \right) \), where the return in \( t \) \( (r_t) \) is equal to the natural logarithm dividing the day’s closing price \( t \) \( (P_t) \) by the day \( t -1 \) \( (P_{t-1}) \).

In order to enhance the reader’s knowledge of the Brazilian market, graphs were developed containing stock market price series and the historical series of returns of the companies studied.

Next, the data windows to be used to obtain the skewness and kurtosis of this data were defined. Considering that an average year has 247 trading days, the following observation windows were defined:
Table 1 – Number of trading days for each data window

<table>
<thead>
<tr>
<th>Window</th>
<th>Number of trading days</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 month</td>
<td>21</td>
</tr>
<tr>
<td>3 month</td>
<td>61</td>
</tr>
<tr>
<td>6 month</td>
<td>123</td>
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<tr>
<td>1 year</td>
<td>247</td>
</tr>
<tr>
<td>3 years</td>
<td>741</td>
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<td>5 years</td>
<td>1235</td>
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<tr>
<td>10 years</td>
<td>2470</td>
</tr>
<tr>
<td>15 years</td>
<td>3705</td>
</tr>
</tbody>
</table>

Once the observation windows had been defined, they were recalculated after every fifteen trading days starting on the first business day in 2011. In this way, each data window’s kurtosis and skewness were calculated on the first trading day of 2011, then on the sixteenth, the thirty-first and so on until the two hundred fortieth trading day, totaling seventeen kurtosis and skewness values for each observation window for each company.

The data relevant to the companies’ stock and options prices were taken from the BM&FBovespa, as were their characteristics (maturity date and price during the exercise) and the risk-free interest curve (in this case, the DI x Pre).

There were cases where it was not possible to directly obtain the risk-free interest rate, since the goal was to obtain the expected rate for a number of trading days Y and this information did not appear in the data supplied by the Stock Exchange. In such cases, Matlab’s cubic spline interpolation method was used to calculate the interest rate for the desired number of days.

To obtain a number of points to formulate the implied volatility graph for the exercise price, five options were chosen; two in the money, one at the money and two out of the money. This number of options was defined taking into account the liquidity of the stocks at issue, with five options being the minimum required to obtain a significant liquidity.

The two in and out of the money options were chosen taking the proximity to the value of the at the money option and high liquidity into consideration. In addition to these criteria applied for choosing the options, those options closest to maturity were also used in the calculation, provided they were not due to expire the following week.

Once the data necessary to calculate each option’s implied volatility had been collected, a VBA formula was created from the Modified Corrado-Su formula and another for the Black-Scholes model.

These formulas sought to estimate the volatility that equaled the call option’s market price iteratively, given its other characteristics and the risk-free interest rate. The purpose of using the iterative method was to calculate two call options prices given their two initial volatilities, a very low one (close to zero) and a very high one (400%). The two volatilities, higher and lower, were then brought closer together, taking into account the difference between market prices and the prices calculated, until the price calculated according to the volatilities converged to the market price. At this point the higher and lower volatility would converge to a single value and this would be the implied volatility. For the system to function, the research accepted an error (difference between the market price and the one calculated iteratively) of 0.000001 Brazilian reals.

Subsequently, the implied volatility graphs were created according to the exercise price for each trading day containing eight curves, one for each observation window. In order to visualize any possible influence of the imminence of maturity, the graphs were analyzed in ascending order by number of days before expiry instead of trading day order.
Lastly, several analyses of the implied volatility curves obtained were undertaken. The first was a comparison of each data window’s slope coefficients, attempting to assess which provided the best performance. The research used the slope coefficient due to the fact that the characteristic of the curves found were more like a smirk than the expected smile.

After this, the implied volatility was calculated using the BS model and its performance was compared to that of the Corrado-Su model.

Concluding, the Wilcoxon Matched-Pairs Signed Rank was applied, which, according to Black (2009), is a non-parametric test used to compare the average of two dependent samples. The data used were not considered independent because they were obtained from the same set of data, the only change being the timeframe used for obtaining kurtosis and skewness. That said, it is worth pointing out that the data windows which generated these two metrics contained similar data, in that the three month window contained the data from the one month window with another two months added and this phenomena is repeated in all other data windows.

This test was carried out to assess if the different data windows and the BS model create implied volatility curves with different slope coefficients, or not. The non-parametric test was chosen because it is more robust and a significance level ($\alpha$) of 5% was used.

4. Results

The first noteworthy point in the results is related to kurtosis and skewness. It became clear that these metrics would vary more within a one year period than long term ones which proved to be leptokurtic while the short term ones were, at times, platykurtic.

The kurtosis obtained from the fifteen year historical data differs from the others, including those calculated using five and ten year periods, because they presented values closer to ten for Petrobras and eighteen for Vale, probably due to the financial crises at the end of the last century.

This extremely long term kurtosis bias ultimately compromised the calculations, which becomes clear when comparing the implied volatility with the other data. This underlines the importance of correctly choosing the historical period to be used when calculating kurtosis and skewness.

Regarding skewness; for the most part it remained between the -1 and 1 data windows, not far removed from normal distribution in this respect, thus proving the larger influence of kurtosis over the distribution of returns.

The second point is the model’s non-adherence at certain times, when it became impossible to calculate the value of implied volatility from the market data. This non-adherence occurred with in-the-money-options only and was not influenced by proximity to maturity.

In most cases, the inability to calculate implied volatility persisted regardless of the window used. In those cases where the model presented implied volatility in some data windows only, the values differ from all others in that they increased the smile in relation to those cases presenting total adherence to the model.

Despite not being the focus of this paper, the cases of non-adherence were analyzed to see if they had a common factor. The only proven characteristic, besides being call options, was the difference between the option’s price and its immediate exercise price which reached, at most, 20%. Even so, it cannot be confirmed that this is a relevant fact, since there were cases with this same characteristic that the model was able to price correctly. It is worth noting that in one case the immediate exercise price exceeded the option’s premium.

As already outlined in the methodology, the Slope Coefficients (SC) of each implied volatility were analyzed in order to test which data windows were capable of smoothing the
volatility smile most. These coefficients are shown in the tables 2 and 3, with the size of the data window indicated next to each of them.

Table 2 – Petrobras Slope Coefficient

<table>
<thead>
<tr>
<th>SC 1m</th>
<th>SC 3m</th>
<th>SC 6m</th>
<th>SC 1y</th>
<th>SC 3y</th>
<th>SC 5y</th>
<th>SC10y</th>
<th>SC15y</th>
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Table 3 – Vale Slope Coefficient

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<th>SC 3m</th>
<th>SC 6m</th>
<th>SC 1y</th>
<th>SC 3y</th>
<th>SC 5y</th>
<th>SC10y</th>
<th>SC15y</th>
</tr>
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<td>-0.0278</td>
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<td>-0.0138</td>
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<td>-0.0470</td>
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<td>-0.0243</td>
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<td>-0.1676</td>
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<td>-0.1356</td>
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<td>-0.0391</td>
<td>-0.0360</td>
<td>-0.0332</td>
<td>-0.1517</td>
</tr>
</tbody>
</table>
The average of the absolute values of the SC was calculated, it was used the absolute value in order to measure the distance from the straight constant, regardless of slope’s direction. The standard deviation (SD) of the original values was also calculated in order to verify the coefficients’ real dispersion. The results are presented in the tables 4 and 5.

Table 4 – Mean and standard deviation of the Petrobras slope coefficients

<table>
<thead>
<tr>
<th></th>
<th>SC 1m</th>
<th>SC 3m</th>
<th>SC 6m</th>
<th>SC 1y</th>
<th>SC 3y</th>
<th>SC 5y</th>
<th>SC10y</th>
<th>SC15y</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.0213</td>
<td>0.0392</td>
<td>0.0264</td>
<td>0.0144</td>
<td>0.0472</td>
<td>0.0404</td>
<td>0.0314</td>
<td>0.1465</td>
</tr>
<tr>
<td><strong>SD</strong></td>
<td>0.0227</td>
<td>0.0692</td>
<td>0.0462</td>
<td>0.0177</td>
<td>0.0368</td>
<td>0.0214</td>
<td>0.0198</td>
<td>0.0521</td>
</tr>
</tbody>
</table>

Table 5 – Mean and standard deviation of the Vale slope coefficients

<table>
<thead>
<tr>
<th></th>
<th>SC 1m</th>
<th>SC 3m</th>
<th>SC 6m</th>
<th>SC 1y</th>
<th>SC 3y</th>
<th>SC 5y</th>
<th>SC10y</th>
<th>SC15y</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.0167</td>
<td>0.0225</td>
<td>0.0374</td>
<td>0.0317</td>
<td>0.0425</td>
<td>0.0295</td>
<td>0.0285</td>
<td>0.2285</td>
</tr>
<tr>
<td><strong>SD</strong></td>
<td>0.0184</td>
<td>0.0317</td>
<td>0.0510</td>
<td>0.0225</td>
<td>0.0202</td>
<td>0.0126</td>
<td>0.0115</td>
<td>0.0495</td>
</tr>
</tbody>
</table>

The results presented here justify the assertion that the fifteen year data window contained a bias which caused a disproportionate volatility smile in relation to the others. Another data window which obtained unsatisfactory performance was the three year window, because in both cases it obtained the second worst mean, despite not having the highest standard deviations.

The poor performance of the three-year data window is probably due to the fact that it is composed mainly of post-2008 crisis data, a moment of great instability in the stock market that was reflected in the pricing of options.

Relevant to the companies used in this study, the data windows which best smoothed the volatility smile and obtained good consistency, low standard deviation, were, in the case of Petrobras, the one year and one month windows respectively, while for Vale the best windows were the one month and the three month windows, despite the fact they did not present the lowest standard deviation.

The one month window’s good performance for both companies suggests that short term returns supply kurtosis and skewness which best adjust returns distribution in the Corrado-Su model to distribute the asset in by reducing the volatility smile.

4.1. Corrado-Su vs. Black-Scholes Models

Considering that in the previous section the data windows which best smoothed the smile were established, the next step was to compare the model applied above with the model traditionally used by the market. In this way, it was possible to assess whether the method applied to smooth the problem was effective or not.

For this purpose, it were presented below (table 6) the slope coefficients obtained from the implied volatility curves generated by the BS model for both companies included in the study, in addition to the mean and standard deviation (table 7), in the same way the data was presented above for the Corrado-Su model.
Upon analysis of the data presented, it can be noted that in the case of Petrobras the performance of the BS model was a little lower than the best data window of the proposed model and was better in the case of Vale.

It is important to note that the BS model did not adhere to all the data, having the same problem that the proposed model had, however this was not able to calculate the implied volatility in fifty percent less cases than the other, showing better adjustment of the data.

### 4.2. Wilcoxon Signed Rank Test

The test was applied in order to assess if there was a statistically relevant difference between the averages of the data or whether they could be regarded as being similar. In this way, it was possible to establish which data windows were similar and which provided statistically divergent results.

As stated above, the test was elaborated considering a 5% level of significance and the slope coefficients were each compared to all other pairs. Tables 8 and 9 present the p-value obtained in each test performed for Petrobras and Vale, respectively.
Table 8 – Wilcoxon Signed Rank Test for the Petrobras slope coefficients

<table>
<thead>
<tr>
<th></th>
<th>SC3m</th>
<th>SC 6m</th>
<th>SC 1y</th>
<th>SC 3y</th>
<th>SC 5y</th>
<th>SC 10y</th>
<th>SC 15y</th>
<th>SC BS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC 1m</td>
<td>0.758</td>
<td>0.687</td>
<td>0.492</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.084</td>
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<tr>
<td>SC 3m</td>
<td>-</td>
<td>0.287</td>
<td>0.381</td>
<td>0.006</td>
<td>0.005</td>
<td>0.006</td>
<td>0.000</td>
<td>0.149</td>
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<tr>
<td>SC 6m</td>
<td>-</td>
<td>-</td>
<td>0.776</td>
<td>0.003</td>
<td>0.005</td>
<td>0.005</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>SC 1y</td>
<td>-</td>
<td>-</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.019</td>
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<td>SC 3y</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>0.379</td>
<td>0.015</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>SC 5y</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
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<tr>
<td>SC 10y</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>SC 15y</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 9 – Wilcoxon Signed Rank Test for the Vale slope coefficients

<table>
<thead>
<tr>
<th></th>
<th>SC3m</th>
<th>SC 6m</th>
<th>SC 1y</th>
<th>SC 3y</th>
<th>SC 5y</th>
<th>SC 10y</th>
<th>SC 15y</th>
<th>SC BS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC 1m</td>
<td>0.356</td>
<td>0.084</td>
<td>0.010</td>
<td>0.001</td>
<td>0.007</td>
<td>0.007</td>
<td>0.000</td>
<td>0.507</td>
</tr>
<tr>
<td>SC 3m</td>
<td>-</td>
<td>0.004</td>
<td>0.031</td>
<td>0.006</td>
<td>0.011</td>
<td>0.015</td>
<td>0.000</td>
<td>0.025</td>
</tr>
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<td>SC 6m</td>
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<td>-</td>
<td>0.449</td>
<td>0.210</td>
<td>0.381</td>
<td>0.407</td>
<td>0.000</td>
<td>0.001</td>
</tr>
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<td>-</td>
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<td>1.000</td>
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<td>-</td>
<td>-</td>
<td>0.002</td>
<td>0.001</td>
<td>0.000</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>0.000</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>SC 15y</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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In the case of Petrobras, we observed that the results obtained with the short term windows were similar, because it was not possible to confirm that their mean are statistically different. Comparing them to the results of the Black-Scholes model, it can be seen that the short term windows, with the exception of the one year window, also obtained similar results.

On the other hand, the long term windows did not obtain any means which could be considered similar, except the three year window with that of five years, due to the p-value of 0.379.

For Vale, the results showed signs of similarity, due to the data window with better performance in the Corrado-Su model, of one month, not having an mean different to the slope coefficient mean of the Black-Scholes model. There was also a similarity between the short term windows, even at a lower level, because the similar pairs were those of the one and three month windows, one and six month windows and one year and six month windows.

Different to Petrobras, the long term windows in certain cases obtained a p-value which did not allow us to affirm that its mean were statistically different from other long and short term windows.

The short term mean, together with the better short term performance seen in the previous analyses, reinforces use of short term data for calculating kurtosis and skew in order to smooth the volatility smile.

In addition, the results point to the performance equivalence between the data window which smoothed the volatility smile most by the Corrado-Su model, which were the short term ones, and the Black-Scholes model.

It is important to point out that only the 15 year data window obtained a different mean to the others, providing evidence for the previous declaration that this window contained a bias.
5. Conclusion

The purpose of this study was to assess which observation windows would generate kurtosis and skewness that would smooth the volatility smile the most in the Corrado-Su model, considering the slope coefficient as an assessment metric.

Upon consideration of the results obtained, the study indicates the best performance was provided by the short term windows (mainly those of one month) over long term windows, since this window obtained a good performance in both studies, which did not occur with the other short term windows.

Considering the depth of the analysis, the implied volatility was calculated using the Black-Scholes model as a measure of control and comparison for the applied model. Once the performance indicator was established, it was possible to follow the analysis of results and conclude that the short term windows in the model proposed obtained similar results to those of the control and that the long term windows did not.

Thus, the results obtained suggest that if the Corrado-Su model is used to calculate implied volatility, the study should use short-term data.

Due to the similar performance of the two models applied, one should encourage the use of the Black-Scholes model given that it requires less market data and thus fewer research and computational resources. However, the person using the pricing tool must evaluate which of the models is considered the most suitable for his/her needs and resources.

In addition to the results outlined above, two other points must be considered. Firstly, the negative influence of a period of financial crisis, such as the one at the end of the 90s and the crisis of 2008, may increase the volatility smile, because it provokes distortions in the kurtosis and skewness calculation which may later impact the pricing models. Secondly, the non-adherence of models to some market data revealed the need for more study regarding this event in order to discover its cause.

6. References


