A theoretical approach to the numismatic market in the United States

Rodrigo de Oliveira Leite
PhD Candidate EBAPE/FGV
rodrigo.leite2015@fgvmail.br

Fabio Caldieraro
Associate Professor of Marketing EBAPE/FGV
fabio.caldieraro@fgv.br

Abstract

Our paper brings a theoretical approach to the numismatic market in the United States. The term “numismatic market” refers to the “collectible coins” market. This market brought a total revenue of $4.8 billion to the US Mint in the 2014-2015 period. The US Mint sell the coins online and uses a network of independent retailers in order to sell the coins physically (“physical stores”). The Mint also sets the price and a “discount rate” to retailers, therefore also setting their profit. Our model showed that the demand for the retailers in this supply chain will be greater than for the Mint. We also show that the Mint will choose the discount rate so that the participation constraint of the retailers is satisfied, in order for the retailers’ profit to be not too small that no retailer would be interested in reselling the coins. Possible research interests for the Brazilian market are also discussed.

Keywords: numismatic market, mathematical modelling, profit optimization strategy.

Introduction

Numismatics is a word derived from the Greek word nounisma, meaning “coin”, and the termination –tics is a Latin adjective, therefore this word means “of coins”. In modern coinage, especially in the United States, it is quite common for the government to issue “numismatic items”. Differently from “usual coins”, these items (usually coins) are not intended for circulations; instead, they are sold to collectors and investors. As an example, the Silver Eagle contains an ounce of .999 Silver, but has a face value of 1 Dollar. However, the Mint usually sell the Silver Eagle around the spot price of Silver at that moment.

The “numismatic market” is extremely large in the United States. According to the Professional Numismatists Guild, it is worth $5 billion dollars not counting the “modern coins” market (Reaney, 2015). This estimation takes only into consideration coins minted in precious metals intended for circulation (pre-1964 in Silver and pre-1933 in Gold). In 2015 alone, the US Mint’s revenue through numismatic items (excluding circulating coins) was $2.5 billion dollars. When we take into consideration 2014 and 2015 together, the total numismatic revenue was $4.8 billion (US Mint, 2016). This estimation does not take into consideration the secondary market of these coins, only the principal market, i.e. the first time they are sold to the public.

Nevertheless, the way that coins are sold vary from country to country. In some countries (eg. Brazil) the Central Bank is the responsible branch of selling the coins, in other countries it is the Mint responsibility (eg. the United States), and other countries sell the rights for certain companies to mint coins in their behalf (eg. Palau). Those differences also make
differences in the sales channels employed by those different agents. In this study we will focus on the US market, and discuss possible research interests for the Brazilian market after the conclusion.

Up to this point, to the best of the authors’ knowledge, there is no study that investigated the dynamics behind the US numismatic market and the relation between the US Mint, retailers and collectors. We bring a theoretical approach to this dynamics using a parsimonious mathematical model. Before the model description and setup we will present a literature review in the subject of numismatics and collecting behavior.

**Literature Review**

Numismatics is a very broad science which receives contributions from others sciences such as economics, sociology, political science, engineering and philosophy (Walker & Morgan, 2015). The selling of commemorative non-circulating legal tender coins is a common trade in the modern world. The coins can be made of precious or non-precious metals, they can also receive a common finish, a matte/satin finish, a brilliant finish or a proof finish (Cuhaj, 2014). Those different types of metals and finishing techniques alters the “perceived value” the collector assigns to a particular coin.

McIntosh and Schmeichel (2004) divided collectors in four subcategories: the passionate, that is usually irrational; the inquisitive, that sees the collection as an investment; the hobbyist and the expressive, to whom a collection is an expression of her identity. Belk (1995) showed that collecting is perceived as a beneficial activity, especially because it is also perceived as “more valued and less selfish than other forms of luxury consumption”. However, if a person develops “erroneous beliefs about the nature of possessions” the collector can escalate to become a hoarder, which is usually perceived as a negative behavior (Frost & Huntl, 1996; Steketee, Frost & Kyrios, 2003).

The “inquisitive collector”, i.e. the collector that is in reality an investor, can derive profits in the secondary market (Frey & Pommerehne, 1989). As an example, there are several examples of successful investments in paintings (Anderson, 1974). However one should be aware that in the “antiquities” the prices paid by the market “can float more or less aimlessly and their unpredictable oscillations”, because the quantity is fixed and the demand fluctuates, as shown by Baumol (1986). Corroborating to this view Burton and Jacobsen (1999) showed that the annual real return on coin investments fluctuated between 13.37% and -8.70% in the 1970-1991 time frame, considering different types of coins.

The act of buying a next item to the collection is the last step in a long process of choosing, selecting, waiting and hunting. As McIntosh and Schmeichel (2004, p.92) explains: “the tension that has been built up in the preceding stages now finds its release in ownership of the item”. The act of possessing a collectible item can be compared to a “mystical experience” (Belk, 1991). In addition, the collector have a “need of completion”: they need to complete the set as fast as possible (Carey, 2008). That is the reason why we usually see luxury goods being sold in physical stores instead of online, the buyer wants to “experience the ownership” of the good right after the purchase, and not waiting until several days later when the good is delivered.

Danet & Katriel (1989) interviewed a total of 165 adult and children who were collectors. They found that the collectors have five different strategies to fulfill this “need for completion”. The first is completing the series, i.e. acquiring all coins of a series; the second is filling a space, such as an album of coins. The third way is creating a visual display, an example of this is a thematic collector that only collect coins that depict boats; the forth is
manipulating the scale, for example: collecting only coins that weight 5 ounces or more. Finally, the fifth way is to “aspire to perfection”, i.e. only collect coins in mint condition.

Dickie, Delorme & Humphreys (1994) established how do a collector perceives the value of a rare coin. She usually look at the denomination (face value), vintage and type (the image struck in the coin and the event it celebrates). In addition, she also looks to which Mint location the coin was produced, the intrinsic value (the value of gold, silver, or any other precious metal in the coin) and the condition of the coin. Translating this definition to modern collectible coins, we can see that the “vintage” factor plays no role, since the Mint issues only modern coins. However, every other factor plays an important role, even the Mint location, which could be San Francisco, Denver, Philadelphia or West Point, all of those branches produce coins in the United States. As an example of the importance of the coin type, the US Mint sold a few gold Kennedy Half Dollar coins (celebrating the 50th anniversary of this coin type) at the 2014 American Numismatic Association Convention at Dallas. The Mint price for this coin was 1,240 dollars (Smith, 2014) and, at that same day, dealers were selling these coins for 5,000 dollars. A week later the first coin sold by the Mint was resold for 100,000 dollars (Gilkes, 2014).

Model description

The Mint. The model presented in this article, was developed in order to replicate the behavior of how collectible coins are sold in the United States. In the US the Congress authorizes the Mint to produce coins. Although some coins have a production limit, most do not have a limit (eg. the $1 Silver Eagle), or the limit is above the demand. Therefore, generally speaking, the Mint is authorized to produce enough coins to supply the market demand. The Mint does not have physical stores, and consumers are only able to buy coins online through the Mint’s website.

The Retailers. In the United States, since the Mint does not sell coins in a physical store, it uses a network of independent retailers that buy the coins with a discount and resell to collectors. Those independent retailers do have physical stores in which the coins can be sold. Since retailers are independent, they are not bound to the Mint by any contract or agreement, they are completely free to choose to buy the coins from the Mint, if they find it profitable, or not, if they foresee that profits are too small, or if they predict that demand for that coin will be low. In addition, retailers are able to buy from the Mint multiple times with different quantities during the production window of that specific coin type.

The Collectors. A collector, therefore has the option of buying online from the Mint, or they have the second option of buying physically from the retailers. Since buying physically the collector have their “need for completion” satisfied immediately, they usually prefer to buy a coin in a physical store (Danet & Katriel, 1989; Carey, 2008).

The Model. We developed a parsimonious model where the Mint $M$ sells the coins through their website directly or through two independent retailers. We use the index $x \in \{M, 1, 2\}$ to index these players (suppliers). In the American model, the Mint has the power to fix the price to consumers and all suppliers price at the same level; thus, the final collector pay the same price $p$ independently of the supplier. The Mint, however, gives an absolute discount of $k$ per coin to the retailers.

Given this, the profit for the retailers is given by

$$\pi_x = [p - (p - k)] \cdot d_x, \quad \text{for } x = \{1, 2\}.$$

By the same token, the profit for the mint is given by
\[ \pi_M = (p - c) \cdot d_M + [(p - k) - c](d_1 + d_2), \]

where \( c \) represents the Mint’s marginal production cost.

We assume that retailers have an outside opportunity valued at \( \pi_0 \). The Mint cannot force retailers to sell the coins unless it provides a profit opportunity for them that is greater or equal to \( \pi_0 \). As it will become clear, this will require that the mint provide a positive discount to the retailers \( (k > 0) \). As a direct result this also implies that the price charged is greater than zero \( (p > 0) \).

The demand

The market demand is derived from the utility maximization of a representative consumer (collector). This collector (representative consumer) derives utility from obtaining a quantity \( d_x \) of coins from supplier \( x \) as follows (the base model adapted from Daughety & Reinganum (2008)):

\[
\begin{align*}
\pi(d_1, d_2, d_M) &= v(d_1 + d_2) + (v - t)d_M - \frac{1}{2}(d_1^2 + d_2^2 + d_M^2) + \\
&\quad b(d_1d_2 + d_1d_M + d_2d_M).
\end{align*}
\]

In the expression above, the term \( d_x \) represents the demand for each supplier, \( t \) captures the disutility that the collector experiences by buying a coin online, and the parameter \( b \) captures the substitution effect between the two retailers and the Mint \( (0 \leq b \leq 1) \). The parameter \( v \) represents the value that the collector perceives from that coin, for model simplicity we will assume that the consumer is rational \( (v \geq p) \), i.e. the collector will not pay more to the coin than she thinks it is worthy.

The collector maximizes her overall utility for consumption by solving the problem:

\[
\max_{d_1, d_2, d_M} \pi(d_1, d_2, d_M) + B - (p \cdot d_1 + p \cdot d_2 + p \cdot d_M),
\]

where \( B \) is her budget. By taking derivatives with respect to \( d_x \) and simultaneously solving for the first order conditions (full proof in the Appendix), we obtain the optimal quantities:

\[
\begin{align*}
d_1 &= d_2 = \frac{v + b(p + t - v) - p}{1 + b - 2b^2}, \\
d_M &= \frac{v + b(p - t - v) - t - p}{1 + b - 2b^2}.
\end{align*}
\]

From the above results, we can already obtain the first conclusions, formalized in Result 1 below.

**Result 1:** The demand for the retailers will be larger than the demand for the mint.

**Proof.** By direct comparison of the demand expressions.

Our model predicts that retailers will sell more coins than the Mint will do. This result comes from the fact that all suppliers set the same price while the consumers experience a disutility from buying online and consequently they will favor buying from the retailers over the Mint. Notice, however, that if the disutility \( t \) was negligible \( (t \to 0) \), the consumer would buy from each supplier indifferently, and thus the suppliers would split the demand equally. However as previous research show (Danet & Katriel, 1989; Carey, 2008), the collectors do experience a disutility \( t \), therefore this result is in agreement with the empirical behavioral reality of the numismatic market.

The suppliers’ equilibrium behavior
To understand the suppliers optimal behavior, we notice first that the retailers do not make any price decision. Their only decisions are to order according to the demand, and whether to participate in the reselling channel or not. Therefore, only the Mint has a real decision power in the market, and we model it accordingly.

Assuming that the Mint wants retailers to participate, then the Mint problem becomes:

$$\max_{p,k} [(p - k) - c](d_1 + d_2) + (p - c)d_M.$$ 

With respect to the following constraints:

$$\pi_x > \pi_0, \ k > 0, \ p > 0, \text{ for } x = \{1, 2\}.$$ 

The solution of the Mint’s problem enables us to establish propositions 1 to 3 below, which will be proven in a single proof.

**Proposition 1:** The first constraint (participation constraint) of $\pi_r > \pi_0$ is the only constraint that is binding.

**Proposition 2:** Price will increase as a function of cost and perceived value, and decrease as a function of the online disutility.

**Proposition 3:** The optimal retailer discount rate $k^*$ will be set so as the participation constraint is satisfied.

**Proof:** In order to solve the above mentioned problem we used the “KKT conditions”, developed by Karush (1939) and Kuhn & Tucker (1951), whose initials gave name to the conditions. The KKT conditions provide a way to solve maximization and minimization problems subject to inequality constraints. Using the KKT conditions we arrive at the following Lagrangian ($L\pi_M$):

$$L\pi_M = \pi_M - \lambda_1 (\pi_0 - \pi_x) + \lambda_2 p + \lambda_3 k.$$ 

Solving the abovementioned problem, we find that the optimal solution occurs when $\lambda_1 > 0, \lambda_2 = 0$ and $\lambda_3 = 0$. Therefore, the first proposition is proven, since the only KKT multiplier greater than zero was $\lambda_1$. That means that the only binding condition is the first one. The complete proof can be seen at the Appendix.

The optimal $p^*$ using the above can be calculated as $\frac{c + v}{2} - \frac{t}{6}$. This means that the price charged by the mint will increase as cost and perceived value increases, but will decrease as the “online disutility” increases, what proves Proposition 2.

Using the same rational from $p^*$, we can show that the optimal discount rate $k^*$ is equal to the following expression:

$$k^* = \frac{6(1 + b - 2b^2)}{3(1 - b)(v - c) + (1 + 5b)t} \pi_0.$$ 

Thus, the Mint will adjust the discount rate $k$ in order to satisfy the participation constraint $\pi_0$. This proves Proposition 3. We can also infer that the discount rate will be set basing in the substitution coefficient, and adjusted considering the perceived value, cost and the “online disutility”.

**Conclusions from the model**
This model provides very valuable insights in the collectible coin markets in the US, and its supply chain. The first result has shown that, under the current division strategy that the Mint sells online and the retailers sell physically, the retailers will sell more coins than the Mint will do. We also highlight the mechanism behind this result: since collectors need to have their “need for completion” satisfied, they will prefer buying a coin physically over buying online, since by buying online they experience the item in the moment, instead of waiting several days to get possession of the coin.

We also prove that the optimal solution for the Mint will be bounded by the fact that the retailers are independent. Because of this fact, the Mint must offer to them a minimum profitability offer: the participation constraint. Proposition 1 proved that this participation constraint is the only one that is binding.

Another quite interesting result is that the optimal price charged by the Mint is $c + v - \frac{t}{6}$, as proved in Proposition 2. This result shows that the price charged by the Mint will be positively affected by both perceived value and cost, and is negatively affected by the “online disutility”. Thus, if the coin has a high perceived value, such as precious metals coins, or commemorating very important events, the optimal price will increase. However, if the collectors’ “online disutility” increases the Mint will charge less for its coins, in an attempt to attract more customers.

Finally, our model shows that the Mint will set its discount rate to the retailer so as the participation constraint is satisfied (Proposition 3). This is a direct effect of Proposition 1, where we have shown that the participation constrain binds the Mint’s strategy. This is important to notice since the Result 1 showed that the physical market represented from the retailers is greater than the online market that the Mint represents. Therefore, it is in the Mint’s best interests to keep the retailers satisfied to have access to this physical market.

These results are quite important for the numismatic market that, according to the Professional Numismatists Guild, worth $5 billion dollars not counting the “modern coins” market (Reaney, 2015). In 2015 alone the US Mint’s revenue through numismatic items (excluding circulating coins) was $2.5 billion dollars. When we take into consideration 2014 and 2015 together, the total numismatic revenue was $4.8 billion (US Mint, 2016). The results from this model highlights the supply chain mechanisms and the price strategy for this quite large market that, up to this point, received no consideration from the marketing science and researchers.

**Further research**

This research analyzed only the modern numismatic market in the United States, however Brazil also produces and sell coins to collectors. As an example, a total of 36 coin types have been produced to celebrate the 2016 Olympics in Rio de Janeiro. This is the greatest numismatic program executed in Brazil by a large margin (in 1900 a total of 4 coin types were produced to celebrate the 4th Centenary of Pedro Alváres Cabral’s arrival to Brazil, and in 1932 a total of 6 coin types were produced to celebrate the 4th Centenary of the São Vicente Settlement).

However, the Brazilian model is quite different from the US, first the retailers control both the physical and the online market, due to the coins being sold by the Brazilian Central Bank (BCB) physically in only a few locations. In addition, the Brazilian Mint is not independent to sell as the US Mint is, it is only a contractor to the BCB that sets the quantity ordered from the Mint and also the price that it will be sold to collectors.
Further research in this area can be useful to compare the differences between the results from the American and the Brazilian strategies in the modern numismatic market. We plan to further extend this mathematical model to englobe the Brazilian market.

Bibliography


Appendix

Complete proof of Result 1

The problem that the collector tries to solve is:

$$\max_{d_1, d_2, d_M} u(d_1, d_2, d_M) + B - (p \cdot d_1 + p \cdot d_2 + p \cdot d_M).$$

Now we calculate the first derivatives (first order conditions), as shown below.

$$\frac{\partial [u(d_i, d_j, d_M) + B - (p \cdot d_1 + p \cdot d_2 + p \cdot d_M)]}{\partial d_1} = v - [p + d_1 + b(d_2 + d_M)],$$

$$\frac{\partial [u(d_i, d_j, d_M) + B - (p \cdot d_1 + p \cdot d_2 + p \cdot d_M)]}{\partial d_2} = v - [p + d_1 + b(d_2 + d_M)],$$

$$\frac{\partial [u(d_i, d_j, d_M) + B - (p \cdot d_1 + p \cdot d_2 + p \cdot d_M)]}{\partial d_M} = v - [p + d_M + b(d_1 + d_2) + t].$$

Now we set these functions (first order conditions) to zero:

$$v - [p + d_1 + b(d_2 + d_M)] = v - [p + d_1 + b(d_2 + d_M)] = v - [p + d_M + b(d_1 + d_2) + t] = 0,$$

by solving the above we arrive at the following demands for the Mint and the retailers:

$$d_1 = d_2 = \frac{v + b(p + t - v) - p}{1 + b - 2b^2}, d_M = \frac{v + b(p - t - v) - t - p}{1 + b - 2b^2},$$

what is exactly Result 1 presented in the paper.

Complete proof of Propositions 1, 2 and 3

We started with the Lagrangian with the KKT multipliers ($\lambda_1, \lambda_2$ and $\lambda_3$):

$$L\pi_M = \pi_M - \lambda_1 (\pi_0 - \pi_r) + \lambda_2 p + \lambda_3 k.$$

Now we solve the Lagrangian setting the first order conditions $\frac{\partial L\pi_M}{\partial p} = 0, \frac{\partial L\pi_M}{\partial k} = 0$ and the KKT conditions $\lambda_1 (\pi_0 - \pi_r) = 0, \lambda_2 p = 0$ and $\lambda_3 k = 0$. There are a total of four “candidate” solutions, that are listed below in Table 1:

<table>
<thead>
<tr>
<th>Solution</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
<th>Price</th>
<th>Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>$\frac{c + v}{2} - \frac{t}{b}$</td>
<td>$\frac{b(1 + b - 2b^2)}{3(1 - b)(v - c) + (1 + 5b)t}\pi_0$</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>$\frac{-2b^2 + b + 1}{b \cdot t + v(b - 1)}\pi_0$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>$\frac{c + v}{2} - \frac{t}{b}$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\frac{v(1 - b) + bt}{1 - b}$</td>
<td>$\frac{[3(b - 1)] \cdot (c - v) + t(1 + 5b)}{2(b - 1)}\pi_0$</td>
</tr>
</tbody>
</table>
The only solution that satisfies all constraints is Solution 1. In Solution 2 we have that $p^* = 0$, what is an impossibility in our model, and Solution 3 suffers the same problem regarding the optimal discount ($k^* = 0$), since the Mint cannot force retailers to have zero profit. For $0 \leq b \leq 1$, the optimal discount rate in Solution 4 is greater than the price ($k^* > p^*$), what is also an impossibility, because the profit function for the Mint is:

$$
\pi_M = (p^* - c) \cdot d_M + [(p^* - k^*) - c](d_1 + d_2),
$$

and in this case $(p - k) - c$ would be negative and, considering that $d_1 = d_2 > d_M$ as shown in Result 1, that would lead $\pi_M$ to also be negative, unless $\frac{d_M}{d_1 + d_2} > 1 - \frac{k^*}{c}$. However, for that to be true $k^*$ must be smaller than the cost, what leads to $c > k^* > p^*$, that also would lead to a negative profit, thus ruling out this solution as feasible.